Geometric Hardening - WARP3D

The Geometric Hardening Fundamentals

Nye Tensor

Incompatability of Deformation Measure

$$\alpha = \nabla \times F^{e^{-1}}$$

where curl $A_{ij} = C_{km} = \varepsilon_{ijk} A_{mj,i}$

The appropriate measure of linear dislocation Density along a slip plane is given by :

$$\lambda^{(s)} = \sqrt{(\alpha \underset{\sim}{n}^{(s)}) : (\alpha \underset{\sim}{n}^{(s)})}$$

where $\underset{\sim}{n}^{(s)}$ is a slip plane normal.

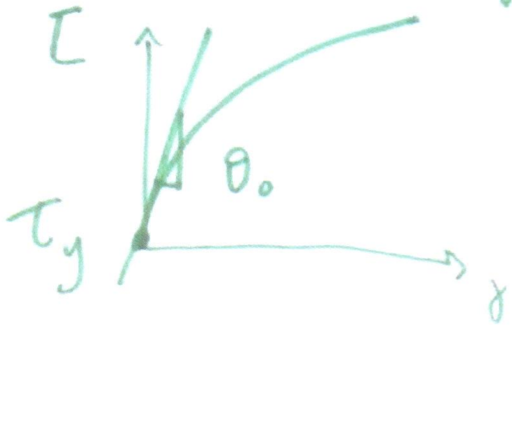
This allows a strengthening of the critical stress on each slip system using

$$\tau_\lambda^{(s)} = \frac{k_0}{k_1}\,\eta\,\mu\,\lambda^{(s)}$$

k_0 : constant characterizing geometric hardening

$k_1 = 2\theta_0 / \eta\mu b$

θ_0 initial slope of $\tau - \gamma$ curve



$\eta = \frac{1}{3}$ from

$\bar{\tau} = \eta\mu b\sqrt{\rho}$

Bailey-Hirsch relation.

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boundaries. The MTS model provides an option to approximate the effect of GNDs – those (theoretically) necessary to maintain compatible deformation. This signed dislocation density contributes to the (forest) work hardening, as the accumulation of immobile dislocations is impacted by their geometric arrangement (in addition to the mean free path storage associated with the statistical density). Acharya *et al.* [1] associate these geometric dislocations with Stage IV hardening. Omitting some derivation and theory (see references for details), the Nye tensor of geometrically necessary dislocations is:

$$\boldsymbol{\alpha} = -\nabla \times \mathbf{F}^{e-1} \; . \tag{3.12.56}$$

The density of necessary dislocations along each slip system is:

$$\lambda^{(s)} = \sqrt{\left(\boldsymbol{\alpha}\mathbf{n}_s^{(s)}\right) : \left(\boldsymbol{\alpha}\mathbf{n}_s^{(s)}\right)} \; . \tag{3.12.57}$$

To include GND effects, Archarya *et al.* suggest an addition to the Voce law to replace Eq. 3.12.48

$$\frac{d\overline{\tau}}{dt} = \sum \theta_0 \left(1 - \frac{\overline{\tau}}{\tau_v} + \frac{\tau_\lambda^{(s)}}{\overline{\tau}}\right)^m \left|\dot{\gamma}^{(s)}\right| \tag{3.12.58}$$

$$\tau_\lambda^{(s)} = \frac{k_0}{k_1}\,\eta\,\mu\,\lambda^{(s)} \tag{3.12.59}$$

where $k_1 = 2\,\theta_0/\left(\eta\,\mu(T)\,b\right)$, $\eta = 1/3$, and k_0 is a parameter (typical values on the order of 1). Computation of the curl of the elastic deformation through an implicit integration is cumbersome; consequently, the computations update values of $\tau_\lambda^{(s)}$ explicitly. The remainder of the hardening evolution remains implicit, as above.

3.12.5.4 Constitutive Model of Ma, Roters, and Raabe

The constitutive model for crystal plasticity developed by Ma and Roters [15] is a physics-based model for face-centered cubic (FCC) materials. The primary hardening variables are the statistically stored dislocation (SSD) density on each slip system, denoted by $\rho_{SSD}^{(\alpha)}$. The total population of dislocations in a material sample is composed of the SSD and GND. The SSD are such that, when averaged over a representative volume, they do not give rise to a residual strain field. In contrast, the GND give rise to residual strain gradient fields and can be linked through the Nye tensor [20] to local (macroscale) curvature of the material to support formulation of gradient based theories. The GND are not currently included in the present constitutive framework; provisions to account for their effects will be treated in future work according to the method presented by Arsenlis and Parks [2]. The SSD density is further decomposed into mobile dislocations $\rho_M^{(\alpha)}$ and immobile dislocations $\rho_I^{(\alpha)}$ such that: $\rho_{SSD}^{(\alpha)} = \rho_M^{(\alpha)} + \rho_I^{(\alpha)}$. The mobile density is typically a small fraction (1-5%) of the total SSD density [15]; expressions for this fraction are provided later.

Additionally, the kinematics of the dislocation motion and also the hardening mechanisms are related to the interactions of dislocations across slip systems. In particular, the model