Monday Sameer is going to present crystal plasticity example.

1. **Latent Hardening (Theory)**
2. Geometric hardening: $k_0$, $k_1$
3. Stress Rate (Objective) $\text{Eqn 3.12.14}$
   - $\text{WARP3D}$
   - Jaumann
Riemann-Asaro-Nedelec

\[ \mathcal{H}_{\alpha\beta} = q_2 \mathcal{H} + (1-q_2) \mathcal{H} \, \mathcal{S}_{\alpha\beta} \]

\[ q_2 = \text{material constant} \]

\[ \frac{1}{Z} \sum_{\gamma}^{N} \mathcal{H}_{\alpha\beta} \gamma^{(\beta)} \]

\[ \mathcal{H}_{\alpha\beta} = \begin{bmatrix} \mathcal{H} & \mathcal{H} \\ \mathcal{H} & \mathcal{H} \end{bmatrix} \]

2-field systems

\[ q_2 = 1 \]
System 2

$\alpha 1$ system

$\gamma$ increasing (isotropic hardening) in 2 same amount as 1, H governs
\[ H_{\alpha \beta} = q H + (1 - q) H \delta_{\alpha \beta} \]

\[ q = \text{const. (idealization)} \]

\[ H(\gamma) = H_0 \ \text{Sech}^2 \left( \frac{H_0 \gamma}{T_s - T_0} \right) \]

\[ = \text{PAN} \]

Changes with \( \gamma \)

\( (H_0, T_0, T_s = \text{current value of shear}, \ T_0, T_s = \text{saturation value}) \)
WARP 3D → MTS model for hardening

\[ T = T_a + T_i + T_y \]

(singular system) (ε, T effects)

Latent Hardening?

\[ LHR = \frac{\text{latent hardening ratio}}{\text{latent system } j_i}, \text{ primary system } i \]
\[ L_{ji} = \frac{T^o_j}{T_{i,\text{max}}} \]

\( T^o_j \) = yield stress of latent system

\( T_{i,\text{max}} \) = max stress reached in the primary system
Primary deformation
- easy glide - only one system is activated
Secondary test - latent system is active interacts with primary system.
$T$ vs $\gamma$ curves for primary & secondary tests (same sample)
Primary Test

\[ \sigma \]

Primary

\[ T \]

Single System

\[ T \text{ (R.S.S.)} \]

\[ \gamma \text{ (R.S. strain)} \]
Rotates the sample

\[ \sigma \]

Secondary

Latent hardening (cross hardening)

\[ T_0 > T_p \]

\[ \max \]

Yield line

Secondary system

\[ L > 1 \]

\[ L = 1 \]

\[ j \geq 1 \]
In real life

\[ H_{\alpha \beta} \quad 12 \times 12 \text{ matrix} \]

In fact

\[ H_{\alpha \beta} \quad 24 \times 24 \text{ matrix (symmetries)} \]

For BCC, HCP it is more complicated

\[ c \]

\[ \begin{pmatrix} c & T \\ -T & c \end{pmatrix} \]

to account for Tension, Compression results are available.
Maurice Berveiller,
Francois Zaoui,

FCC: some results,
BCC: totally understood
HCP: not understood
A to D

A (\text{111})
B (\text{111})
C (\text{1\overline{1}1})
D (\text{1\overline{1}1})

Numbers 1 to 6:

± [011] ± [101] ± [\overline{1}10]
± [0\overline{1}1] ± [\overline{1}01] ± [110]

are slip directions
Stereographic projection has 24 standard triangles each representing one slip system.

1) Thompson tetrahedra
2) Stereographic projection
Fig. 2-9  Standard (001) stereographic projection of poles and zone circles for...
B4 T11
001 011

Stereographic projection

Primary (111)
Self hardening

B4 (111)[101]
(111)[101]
Dislocations in the primary system can interact with conjugate or critical systems to create lower cotblek dislocations. If the resulting dislocation has slip planes that lies on neither one.
of the intersecting planes.
L-C Sessile Lock formation

(D4 primary)

1. A6 \((\Gamma \overline{1}1)\)[110]
2. A6 \((\Gamma \overline{1}1)\)[\overline{1}10]
3. C1 \((\Gamma \overline{1}1)\)[\overline{1}01]
4. C1 \((\Gamma \overline{1}1)\)[0\overline{1}1]

Angle Between two planes:

- \(\beta\) SP slip plane
- [01\overline{1}] (100)
- [\overline{1}10] (001)
- [\overline{1}12] (110)

\(70.5^\circ\)
\(70.5\)
\(109.5\)
\(109.5\)
1) Lower Cottrell Lock (strongest)
2) Hirth Lock

- acts as a barrier C3
- resultant dislocation on a slip plane (third dislocation)

3) \( b_2 \) \( b_3 \) \( b_1 \)
- \( b_3 \parallel b_1 \)
- 

- Glissile Attractive Junction
- Sign is such attractive (force needed to separate from the junction)
- (resultant dislocation on, either slip plane)
\[ B_{4}^{\text{primary}} + \frac{A_2, C_5, D_1, D_6}{\text{secondary}} \Rightarrow \text{finite attractive J und.} \]

1. **Crossslips in system D4**
2. **Coplanar system B2 & B5**
3. **Self hardening in system B4**
(5) Specifying orientation

\[ b_2 \parallel [011] \text{ or } [110] \text{ coplanar} \]

\[ b_1 \parallel \text{ specific (111) plane} \]

(6) Self Healing

B4 \((\text{III}) [\bar{1}01]\)
B4 \((\text{III}) [10\bar{1}]\) 0
180
(4) Crosslip

\[ D4 \ (1\overline{1}1) \ [\overline{1}01] \ 70.5 \]

\[ D4 \ (1\overline{1}1) \ [\overline{1}0\overline{1}] \ 70.5 \]
Further Analysis:

\[ T_j = \sum h_{ji} x_i \]

Primary system active, Step 1:

\[ T_1 = h_{11} x_1 \]

Only \( x_1 \) active

\[ T_2 = h_{21} x_1 \]

\( T_1 \) primary (p), \( T_2 \) is secondary (s)
Step II (\( \chi_2 \) active only)

\[ T_2 = h_{22} \chi_2 + h_{21} \chi_1^{\text{max}} \]

\[ T_1 = h_{11} \chi_1^{\text{max}} + h_{12} \chi_2 \]

active from step I

loading

initial strength from step I

latent hardening
During step II loading,

\[ L_{ij} = \frac{I_L}{I_p} = \frac{h_{22} \delta_2 + h_{21} \delta_1}{h_{11} \delta_1^{max} + h_{12} \delta_2} \]

When \( \delta_2 = 0 \) at beginning Step II, one can evaluate the \( h_{21} / h_{11} \) ratio.
As $\gamma_2$ evolves (if $h_{12}$ is assumed same as $h_{21}$), one can use this equation to solve for $h_{22}$. In fact $L+HR$ increases first reaches a peak. Exp. results

\[
\frac{T_L}{T_p} = \text{known for } \gamma_2 = 0
\]
Imly that $h_{22}$ may not be const. It decreases with applied strain.

\[ h_{22} = q h + (1 - q) S a p L \]

\[ 1 \leq q \leq 1.4 \quad \text{Asaro} \quad q = 1.4 \quad h_{22} \gg h_{11} \]
Jaar van, WAR 3D CP option Monday

1) Latent hard
2) geom. Ko = 0
3) Stress rates (a = )
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