ME 531 – Homework #2 Due February 21, 2018

Patra et al. (see reference below) has proposed that for a single crystal bcc-Fe, the plastic yielding criterion can be expressed as:

$$\tau_{cr} = \mathbf{m}^{(\alpha)} \bullet \sigma \bullet \mathbf{n}^{(\alpha)} + a_1 \mathbf{m}^{(\alpha)} \bullet \sigma \bullet \mathbf{n}_{ns}^{(\alpha)} + a_2 (\mathbf{n}^{(\alpha)} \mathrm{x} \mathbf{m}^{(\alpha)}) \bullet \sigma \bullet \mathbf{n}^{(\alpha)} + a_3 (\mathbf{n}_{ns}^{(\alpha)} \mathrm{x} \mathbf{m}^{(\alpha)}) \bullet \sigma \bullet \mathbf{n}_{ns}^{(\alpha)}$$

where $\boldsymbol{\sigma}$ is the current stress state and $\left\{\mathbf{m}^{(\alpha)}, \mathbf{n}^{(\alpha)}\right\}$ are unit glide vectors corresponding to the slip direction and slip plane normal of the α^{th} slip system respectively. The unit normal $\mathbf{n}_{ns}^{(\alpha)}$ is the non-glide plane normal and makes an angle of $-\pi/3$ radians with the active slip system. For the set of values $\{a_1, a_2, a_3, \tau_{cr}\} = \{0.2407, 0.2564, 0.6028, 157.05\}$ corresponding to 143 K, please construct the yield surface projected on the π plane (namely deviatoric plane) which is described as a plane with a normal parallel to $< \sigma_I, \sigma_{II}, \sigma_{III} >$ such that $\sigma_I = \sigma_{II} = \sigma_{III}$ in principal stress space of $\sigma_I - \sigma_{II} - \sigma_{III}$. Similarly repeat the same calculations for the Schmid law in which $a_1 = 0$, $a_2 = 0$ & $a_3 = 0$, and comment on the differences in between two surfaces.

Hints :

(i) For bcc Fe, potentially, there are 24 slip systems in {110}<111> family distinguishing the slip sense.

(ii) Express $\boldsymbol{\sigma}$ tensor in diagonal form (in principal stress space) employing spherical coordinates such that $\sigma_I = \kappa \sin\theta \cos\phi$, $\sigma_{II} = \kappa \sin\theta \sin\phi$ and $\sigma_{III} = \kappa \cos\theta$ where $\theta \in [0, \pi]$ and $\phi = [0, 2\pi]$. κ is a proportionality coefficient.

Anirban Patra, Ting Zhu, David L. McDowell, Constitutive equations for modeling non-Schmid effects in single crystal bcc-Fe at low and ambient temperatures, International Journal of Plasticity 59 (2014) 1–14