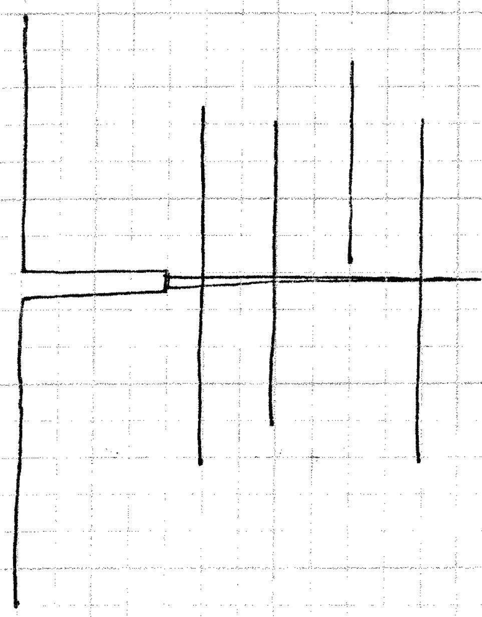
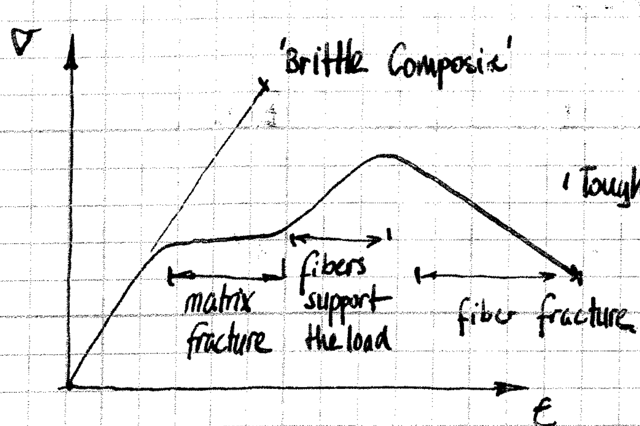
Fracture Behavior of Brittle Composites

04/18/00



Matrix fracture is curtailed by unbroken fiber ligaments in the crack wake. These fibers support the applied load, reduce the stress intensity at the crack tip and increase the overall toughness of composites made from brittle materials.
The terms 'fiber bridging' 'crack bridging' have been used to describe the role of fibers in reducing the driving force for the cracks.



Example

SiC / glass

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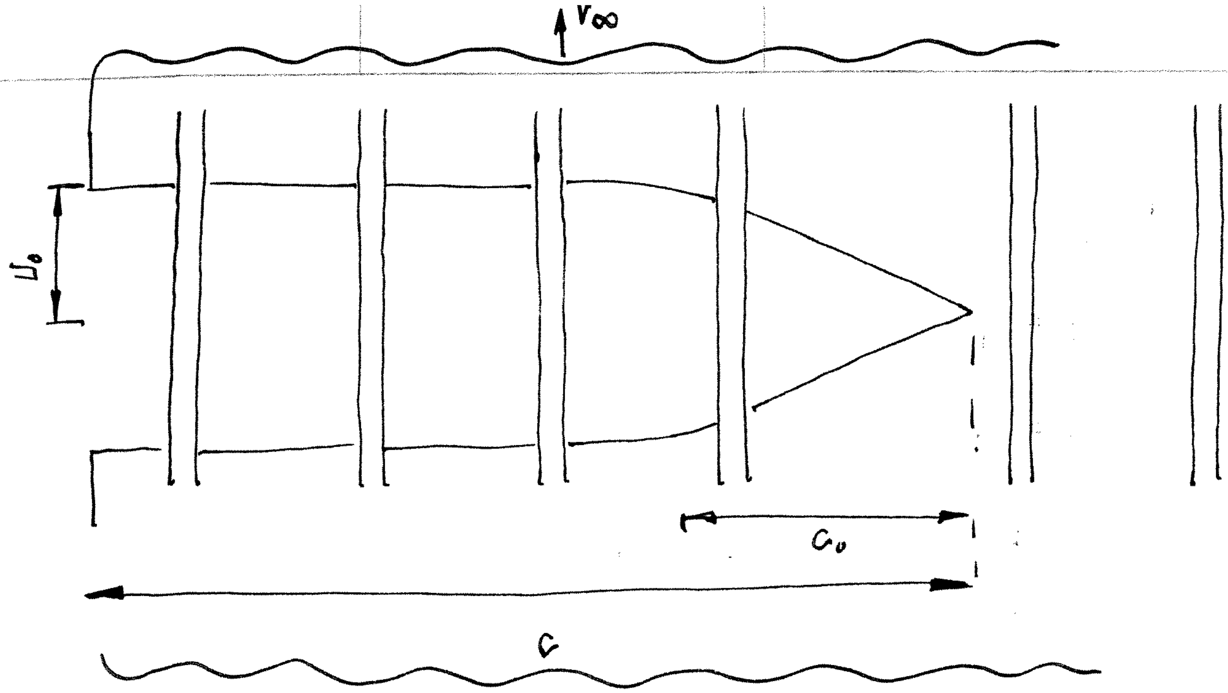
Al_2O_3 . . .

Si_3N_4 . . .



c : total crack length

c_0 : portion of crack over which displacements are below u_0

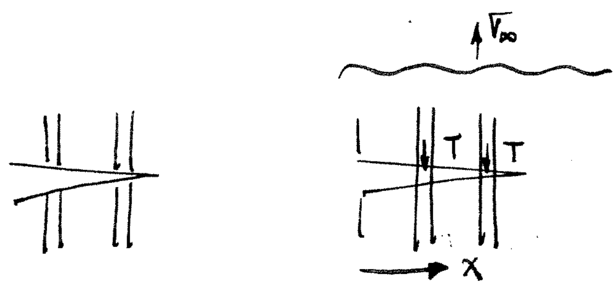
u_0 : steady state (asymptotic) displacement of crack surfaces.

σ_∞ : far field stress (uniaxial)

u : crack tip displacement

(approaches u_0 as $c \gg c_0$)

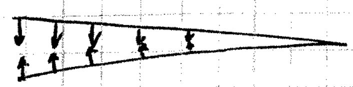
$c \gg c_0$ long cracks. $c \ll c_0$ short cracks



T = traction on fibers that closes the crack

x = distance from the free edge

$X = x/c$ normalized distance

p 

$$p = V_f \cdot \tau(x)$$

$$p = p(x)$$

V_f = vol. fraction of fibers

p = closing pressure on crack surfaces

$$K^L = 2\left(\frac{c}{\pi}\right)^{1/2} \int_0^1 \frac{[\sigma^\infty - p(X)]}{\sqrt{1-X^2}}\, dX \qquad (2a)$$

K^L : composite SIF

If $p(X)=0$ $\quad K^L = \sigma^\infty \sqrt{\pi c}$

Appendix $\quad p = 2\left[u\,\tau\, V_f^2\, E_f\,(1+\eta)/R\right]^{1/2}$

also Eqn. 5 in the Text

u : crack tip displacement

τ : interface shear ~~strength~~ stress

E_f: fiber modulus

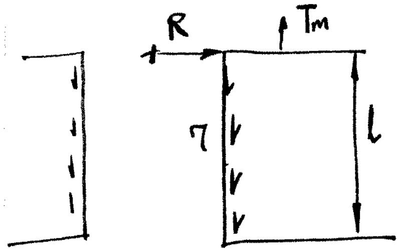
$\eta = E_f V_f / E_m V_m$

R = fiber radius

$$u(X) = \frac{4(1-\nu^2)\,c}{\pi E_c} \int_X^1 \frac{s}{\sqrt{s^2-X^2}} \int_0^s \frac{[\sigma_\infty - p(t)]\,dt}{\sqrt{s^2-t^2}}\,ds$$

s, t : parametric coordinates
ν, E_c : elastic properties of composite

$\sigma_m A_m = 2\pi R l \tau$ (A2) Force equilibrium in matrix alone

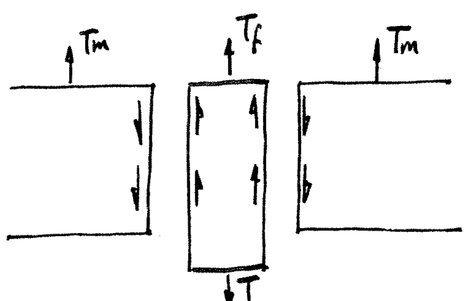


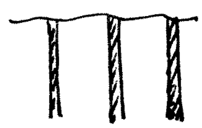
A_m = Area of the matrix
R = fiber radius
l = current sliding distance

Compatibility $\sigma_m / E_m = \sigma_f / E_f$ (A1)

long fibers $\Delta_m = \Delta_f$

$$\frac{\sigma_m}{E_m} = \frac{\sigma_f}{E_f}$$





Force equilibrium for fiber

$$\sigma A_f = 2\pi R l \tau + \sigma_f A_f \qquad \text{(A3)}$$

δ: displacement of matrix from the relaxed configuration due to T_m.

$$\frac{\delta}{l} = [redacted]\pi r\, l\, T / A_m E_m \qquad (A4)$$

[redacted]

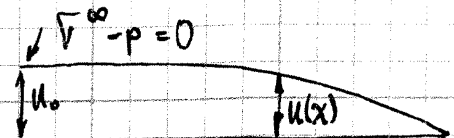
04/20/00

$$T = 2\left[u E_f \tau (1+\eta)/R\right]^{1/2}$$: stress on one fiber

Short crack approximation

At small crack sizes the crack profile does not differ from that of a crack subjected to uniform pressure,

$$u(x) = 2(1-\nu^2) K_L c^{1/2} \left(1 - x^2/c^2\right)^{1/2} / E_c \pi^{1/2}$$



pressure distribution

$$p(x) = \left[\alpha K^L c^{1/2} \left(1 - x^2/c^2\right)^{1/2}\right]^{1/2}$$

where

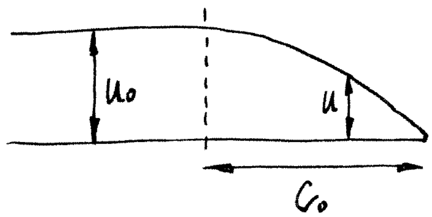
$$\alpha = \frac{8(1-\nu^2)\,\tau\, V_f^2 E_f (1+\eta)}{E_c R \pi^{1/2}}$$

limiting displacement u_0 is given by setting $p = \sigma^\infty$

Limiting displacement $u_0 = \sigma_\infty^2 R / 4 \tau V_f^2 E_f (1+\eta)$

use $u = u_0$ and obtain

$c_0 = \sigma_\infty^4 / \alpha^2 K_L^2$ = transition length



Short cracks $\quad c \le c_0 \quad$ 1.2 edge crack

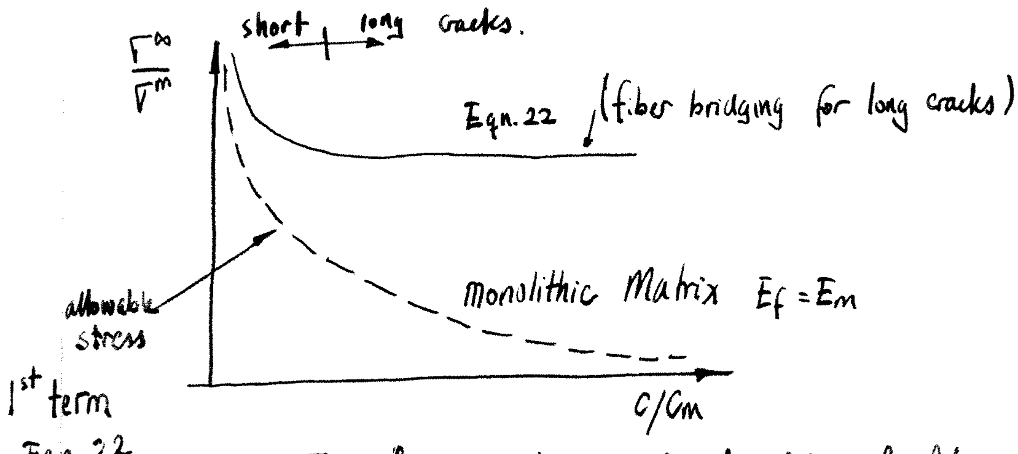
$$K^c = \underbrace{\overset{\sqrt{\pi}}{\Omega}\, \sigma_\infty\, c^{1/2}}_{\text{no bridging}} - \underbrace{\left(\frac{4\alpha}{\pi}\right)^{1/2} K_L^{1/2}\, c^{3/4} \cdot \overset{\text{1.2 edge crack}}{I}}_{\substack{\text{fiber bridging effect} \\ \text{(decreases stress intensity)}}} \qquad (20a)$$

Expressed in normalized form:

$$\frac{\sigma^\infty}{\sigma^m} = \frac{1}{3}\left(\frac{c}{c_m}\right)^{-1/2} + \frac{2}{3}\left(\frac{c}{c_m}\right)^{1/4} \qquad c_m = \left(\pi K_L^c / \alpha I\right)^{2/3}$$

$$\sigma_m = (3/\Omega)\left(K_L^c \alpha I^2 / \pi\right)^{1/3}$$

Eqn. 22 provides a relationship between normalized stress and crack length for fracture to occur.



This figure shows the benefit of fiber bridging on the allowable stress.

Number of assumptions were made

1) 1-D stresses, $T = const.$
2) Parabolic, u profile, actual u may not be parabolic.
3) Theory looks at the wake of the crack not ahead of crack.

04/25/00