ME 530

Homework 1.

20/20 V. Good

Name: Changgong Kim

Due: Sep. 12, 2016

1. (i) Using $\det[a_{ij}] = \varepsilon_{ijk} a_{1i} a_{2j} a_{3k}$, establish the determinant of a_{ij}

$$a_{ij} = \begin{bmatrix} 100 & 50 & 40 \\ 50 & 200 & 30 \\ 40 & 30 & 60 \end{bmatrix}$$

$$\varepsilon_{ijk} = \begin{cases} 1 & ijk = 123, 231, 312 \\ -1 & ijk = 132, 321, 213 \\ 0 & \end{cases}$$

$$\varepsilon_{ijk} a_{1i} a_{2j} a_{3k} = \varepsilon_{123} a_{11} a_{22} a_{33} + \varepsilon_{231} a_{12} a_{23} a_{31} + \varepsilon_{312} a_{13} a_{21} a_{32} + \varepsilon_{132} a_{11} a_{23} a_{32} + \varepsilon_{321} a_{13} a_{22} a_{31} + \varepsilon_{213} a_{12} a_{21} a_{33}$$

$$= 12\times10^5 + 6\times10^4 + 6\times10^4 - (9\times10^4 + 15\times10^4 + 32\times10^4)$$

$$= 132\times10^4 - 56\times10^4 = 76\times10^4$$ ✓

ii) Given σ_{ij}, l_j, determine $\sigma_{ij} l_j = ?$

$$\sigma_{ij} = \begin{bmatrix} 100 & 50 & 40 \\ 50 & 200 & 30 \\ 40 & 30 & 60 \end{bmatrix} \qquad l_j = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$\sigma_{ij} l_j = \begin{bmatrix} \sigma_{11} l_1 + \sigma_{12} l_2 + \sigma_{13} l_3 \\ \sigma_{21} l_1 + \sigma_{22} l_2 + \sigma_{32} l_3 \\ \sigma_{31} l_1 + \sigma_{32} l_2 + \sigma_{33} l_3 \end{bmatrix}$$ ✓

$$= \begin{bmatrix} 100/\sqrt{2} + 50/\sqrt{2} \\ 50/\sqrt{2} + 200/\sqrt{2} \\ 40/\sqrt{2} + 30/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 150/\sqrt{2} \\ 250/\sqrt{2} \\ 70/\sqrt{2} \end{bmatrix}$$ ✓

(iii) Given $C_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}]$

Where λ and μ are constants, determine $C_{1111}, C_{1123}, C_{1122}, C_{3333}$

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

$$\delta_{ij}\delta_{kl} = \begin{cases} 1 & \text{if } i=j \text{ and } k=l \\ 0 & \end{cases}$$

$$C_{1111} = \lambda \delta_{11}\delta_{11} + \mu[\delta_{11}\delta_{11} + \delta_{11}\delta_{11}] = \lambda + 2\mu$$

$$C_{1122} = \lambda \delta_{11}\delta_{22} + \mu[\delta_{12}\delta_{12} + \delta_{12}\delta_{12}] = \lambda$$

$$C_{1123} = \lambda \delta_{11}\delta_{23} + \mu[\delta_{12}\delta_{13} + \delta_{13}\delta_{12}] = 0$$

$$C_{3333} = \lambda \delta_{33}\delta_{33} + \mu[\delta_{33}\delta_{33} + \delta_{33}\delta_{33}] = \lambda + 2\mu$$

2. The 13 elastic constants for the monoclinic phase of NiTi (in the martensitic phase) are given in Table 3 (the B19′ values) from the paper by M.F.-X. Wagner, W. Windl / Acta Materelia 56 (2008) 6232 - 6245. The values are in GPa. Treat all lattice vectors as orthogonal even though the angle between 1 and 3 is slightly higher than 90°

C_{ij} (GPa)	B19′
C_{11}	223
C_{12}	129
C_{13}	99
C_{15}	27
C_{22}	241
C_{23}	125
C_{25}	-9
C_{33}	200
C_{35}	4
C_{44}	76
C_{46}	-4
C_{55}	21
C_{66}	77

$$C_{ij} = \begin{bmatrix} 223 & 129 & 99 & 0 & 27 & 0 \\ 129 & 241 & 125 & 0 & -9 & 0 \\ 99 & 125 & 200 & 0 & 4 & 0 \\ 0 & 0 & 0 & 76 & 0 & -4 \\ 27 & -9 & 4 & 0 & 21 & 0 \\ 0 & 0 & 0 & -4 & 0 & 77 \end{bmatrix}.$$

$$\sigma = C\varepsilon$$

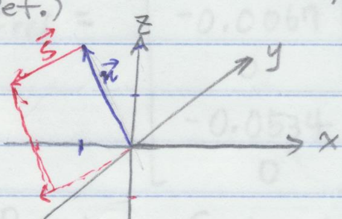
Experimentally, this material has been known to shear on the system listed below. Calculate the shear modulus for the following shear system.

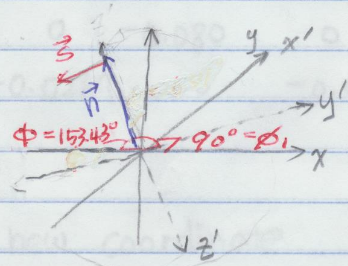
(i) $(\bar{1}02)[\bar{2}0\bar{1}]$

↳ plane normal → direction of shear

① Rotation of coordinate system

Sample Coordinate sys. (X, Y, Z) (Ref.) New Coordinate sys. (x′, y′, z′)





(Bunge)

In terms of euler angles, the rotation can be defined by $\phi_1 = 90°$, $\Phi = \cos^{-1}(-2/\sqrt{5}) = 153.43°$ $\phi_2 = 0$, Then, the rotation tensor can be obtained.

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0.8944 & 0 & 0.4473 \\ 0.4473 & 0 & -0.8944 \end{bmatrix}$$

② Rotate σ from the new coordinate system to the reference coordinate system.

$$\sigma_{ref} = R^{-1} \cdot \sigma \cdot R$$

$(\bar{1}02)[\bar{2}0\bar{1}]$ system is defined as σ_{23} or σ_{32}.

$$\sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma_{23} \\ 0 & \sigma_{23} & 0 \end{bmatrix} \xrightarrow[\text{Rotation}]{} \sigma_{ref} = \begin{bmatrix} 0.8 & 0 & -0.6 \\ 0 & 0 & 0 \\ -0.6 & 0 & -0.8 \end{bmatrix} \cdot \sigma_{23}$$

③ In the reference frame, $\sigma_{ref} = C \cdot \varepsilon_{ref}$.

$$\varepsilon_{ref} = C^{-1} \cdot \sigma_{ref}, \quad \sigma_{ref} = \begin{bmatrix} 0.8 \\ 0 \\ -0.8 \\ 0 \\ -0.6 \\ 0 \end{bmatrix} \cdot \sigma_{23}$$

$$\varepsilon_{ref} = \begin{bmatrix} 0.0176 \\ -0.080 \\ -0.0067 \\ 0 \\ -0.0534 \\ 0 \end{bmatrix} \sigma_{23} = \begin{bmatrix} 0.0176 & 0 & -0.0267 \\ 0 & -0.080 & 0 \\ -0.0267 & 0 & -0.0067 \end{bmatrix} \sigma_{23}$$

④ Rotate ε_{ref} to the new coordinate system.

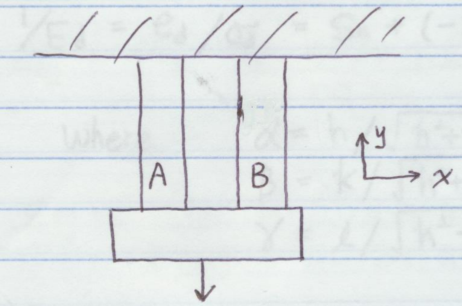
$$\varepsilon = R \cdot \varepsilon_{ref} \cdot R^{-1}.$$

$$\varepsilon = \begin{bmatrix} -0.08 & 0 & 0 \\ 0 & -0.0093 & 0.0263 \\ 0 & 0.0263 & 0.0202 \end{bmatrix} \cdot \sigma_{23}$$

(5) Find $\sigma_{23} / \gamma_{23}$

$$\frac{\gamma_{23}}{2} = 0.0263\,\sigma_{23} \Rightarrow \frac{\sigma_{23}}{\gamma_{23}} = \frac{1}{2} \times \frac{1}{0.0263} = 19\ \text{GPa}$$

3. A hypothetical composite material is composed of two fcc grains of iron as shown below.

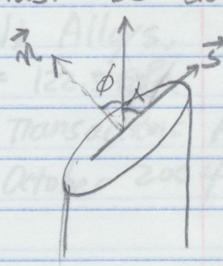


A : [1 0 0]
B : [1 1 1]

The composite is stretched under isostrain conditions in the y direction. Note that the elastic moduli of the two grains are not the same. The critical resolved shear stress of the fcc iron is 40 MPa. As the applied load is increased which bar will yield first? Determine the stresses in grain A and B in the y direction when first yielding is detected.

① Schmid factors of 12 slip systems must be determined for each grain.

Schmid factor $m = \cos\phi \cos\lambda$



	A	B
a. (111)[1 $\bar{1}$ 0]	$1/\sqrt{3} \cdot 1/\sqrt{2}$	0
b. [1 0 $\bar{1}$]	$1/\sqrt{3} \cdot 1/\sqrt{2}$	0
c. [0 1 $\bar{1}$]	0	0
d. ($\bar{1}$11)[1 0 1]	$-1/\sqrt{3} \cdot 1/\sqrt{2}$	$1/3 \cdot 2/\sqrt{6}$
e. [0 1 $\bar{1}$]	0	0
f. [1 1 0]	$-1/\sqrt{3} \cdot 1/\sqrt{2}$	$1/3 \cdot 2/\sqrt{6}$
g. (1$\bar{1}$1)[$\bar{1}$ 0 1]	$1/\sqrt{3} \cdot -1/\sqrt{2}$	0
h. [0 1 1]	0	$1/3 \cdot 2/\sqrt{6}$
i. [1 1 0]	$1/\sqrt{3} \cdot 1/\sqrt{2}$	$1/3 \cdot 2/\sqrt{6}$
j. (11$\bar{1}$)[1 $\bar{1}$ 0]	$1/\sqrt{3} \cdot 1/\sqrt{2}$	0
k. [1 0 1]	$1/\sqrt{3} \cdot 1/\sqrt{2}$	$1/3 \cdot 2/\sqrt{6}$
l. [0 1 1]	0	$1/3 \cdot 2/\sqrt{6}$

m_{max} for $A = 1/\sqrt{6} = 0.4082$

for $B = 2/3\sqrt{6} = 0.2772$

↓

In isostress condition, "A" will yield first.

↓

In isostrain condition, E must be considered.

② Elastic moduli at [1 0 0] and [1 1 1] must be considered.
From the lecture note,

$$1/E_d = e_d / \sigma_d = S_{11} + (-2S_{11} + 2S_{12} + S_{44})(\beta^2\gamma^2 + \gamma^2\alpha^2 + \alpha^2\beta^2)$$

where
$$\alpha = h/\sqrt{h^2+k^2+l^2}$$
$$\beta = k/\sqrt{h^2+k^2+l^2}$$
$$\gamma = l/\sqrt{h^2+k^2+l^2}$$

$d = [1 0 0] \rightarrow \alpha = 1,\ \beta = \gamma = 0$

$$1/E_{[100]} = S_{11} = e_{[100]} / \sigma_{[100]}$$

$d = [1 1 1] \quad \alpha = 1/\sqrt{3} = \beta = \gamma$

$$1/E_{[111]} = S_{11} + (-2S_{11} + 2S_{12} + S_{44})\left(\frac{1}{9} + \frac{1}{9} + \frac{1}{9}\right)$$
$$= S_{11} + (-2S_{11} + 2S_{12} + S_{44}) \cdot \frac{1}{3} = e_{[111]} / \sigma_{[111]}$$

For elastic constants of FCC FeCrNi Alloys,
$C_{11} = 210.9$ GPa $C_{12} = 140.3$ GPa $C_{44} = 122.5$ GPa
(A. TEKLU, Metallurgical and Materials Transactions A, Vol 35A, October 2004)

Using matlab, $S_{11} = 0.0101$ 1/GPa
$S_{12} = -0.004$ 1/GPa
$S_{44} = 0.0082$ 1/GPa

$E_{[100]} = 1/S_{11} = 99$ GPa
$E_{[111]} = 3/(S_{11} + 2S_{12} + S_{44}) = 291$ GPa

③ Find the yield strains

$E_{[100]} = 99\,GPa$ $\quad$ $E_{[111]} = 291\,GPa$

crss, schmid factor

$$\varepsilon_{Ay} = \frac{\sigma_{Ay}}{E_{[100]}} = \frac{40/0.4082\ MPa}{99\ GPa} = 0.00099$$

$$\varepsilon_{By} = \frac{\sigma_{By}}{E_{[111]}} = \frac{40/0.2772\ MPa}{291\ GPa} = 0.00050$$

$\therefore\ \varepsilon_{By} < \varepsilon_{Ay}$. Grain B yields first at iso strain condition

When the grain B yields, the grain A is at 49.5 MPa while the grain B is under 145.5 MPa.

$$\sigma_{By} = \tau_{crss} / m_{max} = 40/0.2772 = 145.5\,MPa$$

$$\sigma_A = 99\,GPa \times 0.0005 = 49.5\,MPa.$$